On the Construction of Ovals

Paul L. Rosin
School of Computer Science
Cardiff University
Cardiff, CF24 3XF, UK
Paul.Rosin@cs.cf.ac.uk

Abstract

For thousands of years there has been the need to construct ovals! Some of the various approaches available to the artist are discussed (pen and string, mechanical devices, etc.). The shapes of several oval pictures are then analysed using the least squares fitting of elliptical and oval models.

1 Introduction

This article looks at the problem for the artist and draughtsman of drawing ovals. In general, a convenient means of drawing geometric forms is desirable, and with straight sided figures this is relatively straightforward (although shapes such as pentagons require some ingenuity). In addition, not only can circles be drawn with compasses, but since the Renaissance many three legged compasses have been designed to draw ellipses and the other conic sections [17].[1]

Compared to ellipses, triangles, rectangles, etc. the definition of ovals is rather vague.[2] Dictionaries generally state that ovals correspond to an egg shape, but of course eggs come in a variety of forms as D'Arcy Wentworth Thompson [23] noted.[3] Thus the term oval covers different degrees of elongation, asymmetric tapering, squareness, etc. Of course, there are many mathematical definitions of ovals (e.g. Cassinian ovals, Cartesian ovals, etc.) and they have been intensively studied.[4] A more recent means of generating ovals is to generalise the concept of the ellipse. Wheres the ellipse has two foci, allowing for three of more foci yields the exotically named n-ellipse [20], egglipse [19], polyellipse [15], etc.[5]

However, rather than such mathematical descriptions, the artist requires a more practical means of drawing the oval. A modification of the common gardener's method for drawing ellipses is available for some Cartesian ovals [4, 7], but involves multiple loops of string attached to the pencil point and is tricky (the string falls off, or sticks...). In contrast to the substantial activity[6] in designing ellipsographs in the last few centuries [2, 9] there was less attention paid to oval drawing devices. There are several approaches; one is to devise mechanical devices consisting of complex assemblies of cogs and wheels, sliding bars, chains, pulleys, etc. similar in conception to the ellipsographs. In this vein William Ford Stanley [21] designed and sold an *oograph*, namely an instrument designed "for an oologist for drawing eggs of birds in their natural sizes and proportions, for which it answers very well". Its construction is explained in figure 1a, and essentially consists of two bars joined at point $\mathbf{A}$. The bar $\mathbf{OA}$ rotates about $\mathbf{O}$ while the other, containing the pen at a selected point $\mathbf{P}$, is constrained to slide along a fixed point $\mathbf{B}$. If the length of the rotating bar $\overline{\mathbf{OA}}$ is set to unity, $\mathbf{B} = (b, 0)$, and the distance between the joint and the pen $\overline{\mathbf{AP}} = L$, then the path of the pen is $\mathbf{P} = \{\cos\theta, \sin\theta\} + L\{b - \cos\theta, -\sin\theta\}/\sqrt{1 + b^2 - 2b\cos\theta}$.

An alternative is provided by Gray [9] which he implies is the standard approach since he says it is an "ove …constructed in the usual way". It also has the rotating bar; see figure 1b. This time however the end of the bar $\mathbf{C}$ is constrained to slide along the horizontal axis. Thus, if again the length of the rotating bar is unity, the

[1] Research in the last fifty years now points to Leonardo da Vinci as the (re)inventor of the ellipsograph [12].

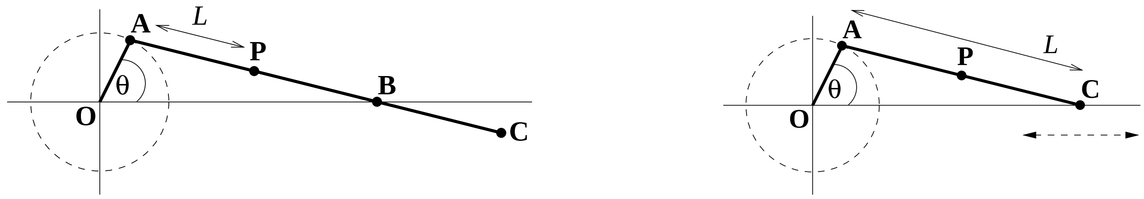
[2] Ellipses are *not* the same as ovals, although Dürer was apparently confused on this issue [11, 16].

[3] "Some few eggs, such as those of the owl or of the tortoise, are spherical or very nearly so; a few, such as the grebe's or the cormorant's, are approximately elliptical, with symmetrical or nearly symmetrical ends; the great majority, like the hen's egg, are ovoid, a little blunter at one end then the other; and some, by an exaggeration of this lack of antero-posterior symmetry, are blunt at one end but characteristically pointed at the other, as is the case in the egg of the guillemot and puffin, the sandpiper, plover, and curlew."

[4] James Clerk Maxwell's first paper [3] (at the age of 15) was on the properties and construction of Cartesian ovals.

[5] The n-ellipse etc. are defined as the locus of points whose summed distances to the multiple foci is constant.

[6] Stanley [21] says "the construction of instruments for striking ellipses seems to have been a favourite theme for exercising the fancy of inventors."



(a) (b)

Figure 1: Instruments for drawing instruments, as described by Stanley (left) and Gray (right).

sliding bar is of length $\overline{\mathbf{AC}} = L$, and the pen at $\mathbf{P}$ is positioned at a proportion w up the bar (i.e. $\frac{\overline{\mathbf{PC}}}{L} = w$), then $\mathbf{P} = \{\cos\theta + (1-w)\sqrt{L^2 - \sin^2\theta}, w\sin\theta\}$.

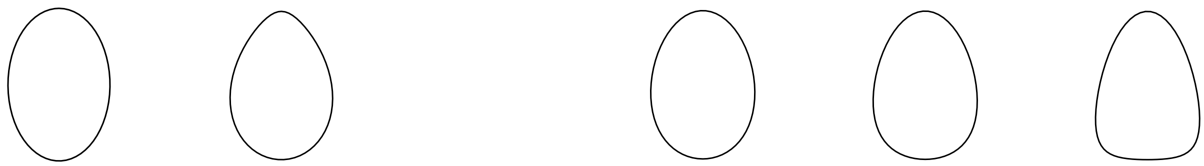
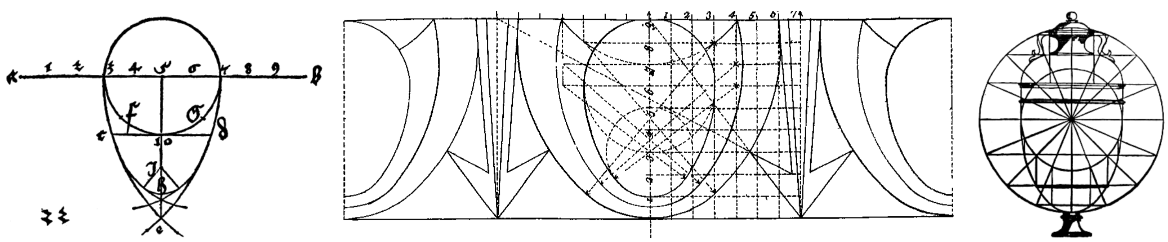


Figure 2: Examples of ovals generated using the instruments of Stanley (left) and Gray (right).



(a) (b) (c)

Figure 3: Examples of ovals constructed using circular arcs: (a) Dürer, (b) Gibbs, (c) Serlio.

Rather than draw ovals according to a single mathematical formula another approach is to simplify the problem by breaking the oval into several more easily drawable pieces. For instance, the gardener's method for drawing ellipses can be modified by looping the string around three pins (instead of two) forming an isosceles triangle. This generates an oval comprised of six portions of ellipses [7]. Even simpler, and therefore more popular, are the many constructions made by joining circular arcs. Like the previous method they also tend to have continuous tangents, which ensures that they look attractive. There is a long history of this approach – Thom [22] claimed that such constructions were used 5000 years ago by Megalithic man for making stone circles, and describes two such types of egg shaped ovals. Since then they have been used for everything from classical egg and dart ornamental architectural moulding (as outlined in James Gibbs' [8] manual), designing the belly of lutes by Arnaut in the fifteenth century [14], as a basis for the design of vases in Serlio's architectural manual [13, 10], to the design of Stradivarius violins [5].

2 Fitting Ovals to Frames

We will finish by focusing on piecewise circular oval approximations to ellipses. These were used extensively in architecture – being first published by Serlio [10] whose four methods of construction are illustrated in figure 4; many more constructions have been published since [18]. We will analyse some ovals in art to determine how they were formed, and to simplify matters, will restrict attention to the borders of the pictures, since they clearly defined.[7]

For each of the following pictures the border has been extracted, the best fit ellipse and ovals determined numerically according to the least squares criterion, and overlaid on the image. In the first example (figure 5) the shape of the inner contour of the frame is clearly seen to be non-elliptical. The best fit ellipse narrows too

[7] Regarding oval frames Arnheim emphasises their playful nature, and notes that they encourage rounded, curvilinear shapes in the picture [1]. Moreover, he suggests that the ellipse's two foci naturally aid compositions containing two centres of attraction.

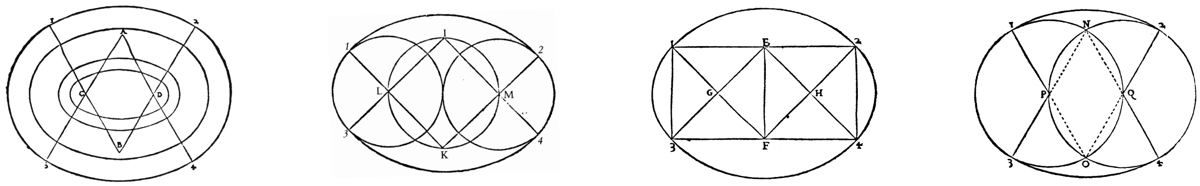


Figure 4: Sebastiano Serlio's oval constructions (1537–1575).

Figure 5: Nicholas Hilliard – The artist's wife Alicia (1578).

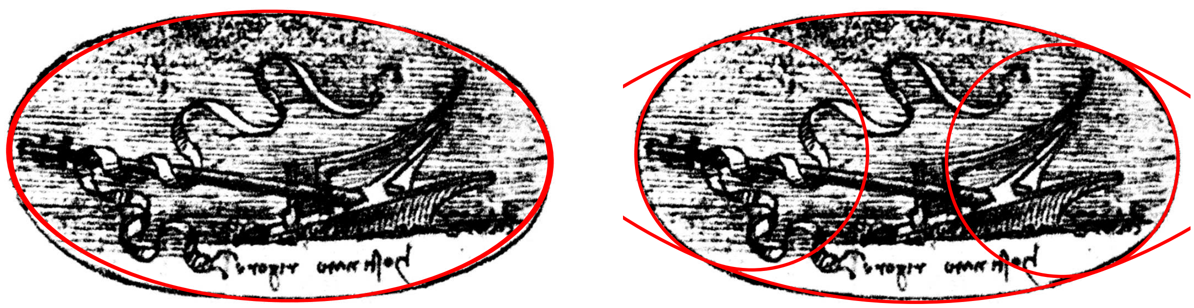


Figure 6: Leonardo da Vinci – emblem with motto *Hostinato rigore*.

Figure 7: George Catlin – "The artist at work painting Indians on the plans" (1857–69)

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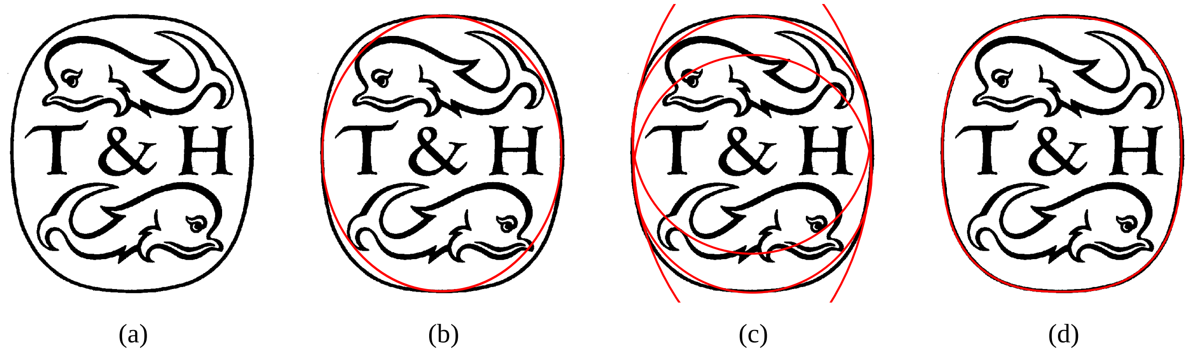

Figure 8: Thames and Hudson logo. (b) the best fit ellipse to the frame, (c) the best fit oval, and (d) the best fit superellipse.

quickly at the ends, but the circular arc oval construction fits very well. The second example shown in figure 6 is just a freehand sketch by Leonardo da Vinci, but yet again the circular arc oval construction fits the hand-drawn outline of the outer contour much better than an ellipse. The third example (figure 7) demonstrates not only that the shape of the painting was constructed using circular arcs, but that the arcs also played a part in the composition of the painting. It can be seen that the circles coincide with the two principal characters in the picture. Finally, a more modern example demonstrates that, given the rather square oval shape, neither the ellipse nor the constructions using four arcs are appropriate. Instead, a superellipse is found to match almost exactly. The superellipse was popularised in the computer age [6], and to the author's knowledge no methods for manual/mechanical construction of superellipses have been devised!

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